SETTLEMENTS IN SANDS DURING EARTHQUAKES

by

W.D. Liam Finn[1] and P.M. Byrne[2]

Introduction

Earthquakes can cause considerable settlements in deposits of unsaturated cohesionless soils. In the San Fernando earthquake of February 9, 1971, settlements of 4in to 6in (approx. 0.10-0.15m) have been reported under a building on spread footings on a 40 ft (12.12m) deep sand fill (Seed and Silver, 1972). Settlements of up to 2ins (0.05m) were noted in other areas after the same earthquake.

Ground settlements resulting from ground shaking during earthquakes are rarely uniformly distributed and cause differential settlements in structures. These differential settlements can be a major cause of damage to structures. The severe damage to some major structures in Skopje during the earthquake there in 1963 was considered to result from differential settlements caused by the compaction of pockets of loose sand under the foundations (Seed and Silver, 1972).

A number of major studies have confirmed the key role played by differential settlements in structural damage. Skempton and MacDonald (1956) related damage to the angular distortion of a building. The angular distortion is defined as the ratio of the differential settlement,

---

1 Professor and Dean, Faculty of Applied Science, University of British Columbia, Vancouver.

2 Assistant Professor, Department of Civil Engineering, University of British Columbia, Vancouver.

Δ, and the distance L between two points after eliminating the effect of tilt. They suggested that the limiting value of Δ/L to cause cracking in walls and partitions is 1/300 and that values of Δ/L greater than 1/150 would cause structural damage.

Polshin and Tokar (1957) described guidelines similar to those cited above which were adopted in the Building Code of the U.S.S.R. in 1955. One particular difference is that they do not eliminate tilt before computing the ratio Δ/L.

Settlements of buildings on sands under static loads is estimated empirically and at the present time,in most cases,little consideration is given to possible additional settlements due to ground shaking during earthquakes. Two semi-empirical approaches have been advanced for estimating earthquake settlements. Seed and Silver (1972) suggested a procedure for estimating settlements in unsaturated sands. Lee and Albaisa (1974) have suggested an empirical procedure for estimating settlements in saturated undrained sands.

Mechanism of Settlements

Settlements in cohesionless soils result from volume compaction. During an earthquake the compaction is caused almost entirely by the dynamic shear strains resulting from the horizontal acceleration components of the earthquake. Research has shown that vertical accelerations and associated dynamic normal stresses are very ineffective in inducing compaction. D'Appolonia (1968) found that vertical vibrations caused little compaction even in very loose fine sands with no surcharge until the vertical acceleration reached about 1g. The application of surcharge to the sand raised the level of acceleration required to cause significant compaction. Whitman and Ortigosa (1969) carried out similar tests and concluded that vertical accelerations during earthquakes caused very little compaction.

The horizontal accelerations during earthquakes generate dynamic shear stresses and shear strains which are very effective in causing volume changes even in dense sand. The behaviour of dry sand under cyclic shear strains has been studied by Youd (1970), Silver and Seed (1971), and Seed and Silver (1972). From these studies it may be concluded that the settlement of sand during earthquakes is caused mainly by the dynamic shear strains. The studies also show that the total settlement caused by a given number of uniform shear cycles depends only on the magnitude of the shear strain cycle.

A detailed study of volume changes in sand caused by cyclic shear strains has been presented by Martin, Finn and Seed (1975). Their results show that the increment in volumetric strain, γ, depends on the magnitude of γ and the total volumetric strain, ε_{vd}, accumulated during the previous cycles of shear strain. The volumetric strain increment may be expressed analytically by

$$\Delta\varepsilon_{vd} = C_1(\gamma - C_2\varepsilon_{vd}) + C_3\varepsilon_{vd}^2 + C_3\varepsilon_{vd}^2/(\gamma + C_4\varepsilon_{vd}) \qquad ..... (1)$$

The sand used in the study was Crystal Silica No. 20 having a D_{10} = 0.5mm and a uniformity coefficient of 1.5. For this sand at a relative density D_r = 45% the constants are C_1 = 0.80, C_2 = 0.79, C_3 = 0.45, and C_4 = 0.73 when the strains are expressed in percentages. At other relative densities the volumetric strain may be calculated from the equation

$$\Delta\varepsilon_{vd} = R(\Delta\varepsilon_{vd})_{45} \qquad ..... (2)$$

At D_r = 60%, R = 0.54; at D_r = 80%, D_r = 0.19. As an independent check of equations (1) and (2) the predicted volumetric strains after 10 cycles of shear strain of various magnitudes are compared with those measured by Seed and Silver (1972) on the same sand. The data are shown in Figure 1 and indicate a satisfactory prediction of volumetric strain for practical purposes.

During an earthquake a sand deposit is subjected to a train of non-uniform shear strains. The procedure outlined above for calculating the volumetric strains has been shown by Martin, Finn and Seed (1975) to be applicable to a sequence of non-uniform strain cycles.

Therefore the distribution of volumetric strains in a sand deposit can be computed provided the shear strain histories at various levels in the deposit can be computed. In horizontal layers of sand the volumetric strains are equal to the vertical strains and from the latter the settlements may be computed.

Calculation of Settlements

A stratum of unsaturated sand resting on horizontal bedrock is shown in Fig. 2. The stratum is of thickness, H, and the properties of the sand are allowed to vary in the vertical direction only. Shaking is due to shear waves propagating upwards from the rock base. Under these assumptions, the dynamic response becomes that of a one-dimensional shear beam.

Since the layer properties may vary vertically in a random manner the shear beam can be approximated by the discrete mass model shown in Fig. 2. The masses are connected by appropriate springs and dampers. The stress-strain characteristics of sands are non-linear, hysteretic and strain dependent. These characteristics are introduced into the model by using the equivalent linear strain dependent shear moduli and viscous damping ratios defined by Seed and Idriss (1969) and shown in Fig. 3. The elemental spring stiffnesses, k_i, are based on the current moduli, G_i, at the middle of each layer of thickness, h_i; $k_i = G_i/h_i$.

The shear modulus G at any time is given by

$$G = 1000\ K_2(\sigma_m')^{\frac{1}{2}} \qquad \text{..... (3)}$$

in which σ_m' is the mean effective normal stress and K_2 is a parameter which varies with strain and relative density as shown in Fig. 3.

The equations of motion of the discrete mass system are given by

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\ddot{u}_g(t) \qquad \text{..... (4)}$$

in which [M] is the diagonal mass matrix, [C] the damping matrix, [K] the stiffness matrix, $\ddot{u}_g(t)$ the base rock acceleration and $\{x\}$, $\{\dot{x}\}$, and $\{\ddot{x}\}$ are the displacement, velocity and accelerations of the masses [M]. Since an equivalent linear method of analysis will be used, the equations are uncoupled by the transformation

$$\{y\} = [\phi]^T[M]\{x\} \qquad \text{..... (5)}$$

in which $[\phi]$ is such that

$$[\phi]^T[M][\phi] = [I] \qquad \text{..... (6)}$$

$$\{\phi_i\}^T[C]\{\phi_i\} = 2\lambda_i\omega_i \qquad \text{..... (7)}$$

and

$$\{\phi_i\}^T[K]\{\phi_i\} = \omega_i^2 \qquad \text{..... (8)}$$

where λ_i = damping in the i^{th} mode and ω_i = the i^{th} natural frequency. The equations then reduce to normal mode equations

$$\ddot{y}_n + 2\lambda_n\omega_n\dot{y}_n + \omega_n^2 y_n = -\{\phi_n\}^T[M]\ddot{u}_g(t) \qquad \text{..... (9)}$$

An iterative procedure is used for solving equations (9). Initital values for shear modulus and damping ratio are assumed to be those for a strain, $\gamma = 10^{-4}$. The response of the system to the prescribed base motion is then computed over a short time interval Δt assuming elastic behaviour. The distribution of average shear strains within the soil profile is determined. If these strains differ by more than an arbitrarily prescribed amount from the strains assumed at the beginning of the analyses, new values for the moduli and damping are selected and the system of equations again solved. The procedure is repeated until two consecutive sets of strains are obtained which agree within the prescribed tolerances. At this stage strain compatible stiffnesses and damping ratios have been achieved.

Different compatible stiffnesses and damping ratios will apply in the next time interval. It is important to ensure that dynamic equilibrium is maintained in the transition from the end of one time interval to the beginning of the next. The final velocities and displacements of the previous time interval become the initial velocities and displacements of the next but because of the abrupt changes in the stiffness and damping there will be a step change in the acceleration. The initial acceleration in the next time interval is given by

$$[M]\{\ddot{x}\}_1 + [C]_1\{\dot{x}\}_1 + [K]_1\{x\}_1 = [M]\{\ddot{x}\}_2 + [C]_2\{\dot{x}\}_1 + [K]_2\{x\}_1 \quad \ldots\ldots (10)$$

in which the subscript 1, indicates "before modification of properties", and subscript, 2, indicates after modification. Therefore

$$\{\ddot{x}\}_2 = \{\ddot{x}_1\} + [M]^{-1}[[[C]_1 - [C]_2]\{\dot{x}_1\} + [[K]_1 - [K]_2]\{x\}_1] \quad \ldots\ldots (11)$$

in which $\ddot{x}_2$ is the acceleration after the change in stiffness and damping in each iteration. The damping matrix for use in equation (11) is conveniently constructed as follows; transformation to normal co-ordinates requires that

$$[\phi]^T[C][\phi] = \begin{bmatrix} 2\lambda_1\omega_1 & 0 & 0 \\ 0 & 2\lambda_2\omega_2 & 0 \\ 0 & 0 & 2\lambda_n\omega_n \end{bmatrix} = [A] \quad \ldots\ldots (12)$$

in which $\lambda_1 = \lambda_2 = \lambda_n$

$$[C] = [\phi^T]^{-1}[A][\phi]^{-1} \quad \ldots\ldots (13)$$

or

$$[C] = [M][\phi][A][\phi]^T[M] \quad \ldots\ldots (14)$$

In the case of saturated sands from which drainage is impeded the pressures in the porewater will increase during shaking. The increase in porewater pressure reduces the mean effective normal stress and changes the value of G. In this case a more complicated analysis must be carried out. This analysis has been described by Finn and Byrne (1975). The analysis shows that the response of saturated sands in the drained and undrained state are almost identical until the porewater pressure rises above 30% of the overburden pressure. For higher porewater pressures settlements can be computed but at such higher water pressures there is more concern about a possible foundation failure or complete liquefaction than with settlements. Consequently in this paper data for the undrained case is not given. Thus $\{x\}, \{\dot{x}\}$, and $\{\ddot{x}\}$ at the beginning of the next time interval are now known and the integration proceeds as before in normal coordinate space. In contrast to current practice for equivalent linear analysis (Seed and Idriss, 1969) the soil properties are progressively modified as the analysis proceeds.

Results

Settlements in a sand layer 50ft (15.2m) thick were computed for a range of earthquake accelerations. The relative density of the sand varied from D_r = 45% to D_r = 80%. The first 10 seconds of the N-S acceleration component of the El Centro (1940) earthquake was used as input motion at the base of the sand layer. This earthquake record was scaled by factors of 0.5, 1.0, and 1.5 to provide three different acceleration histories with maximum accelerations of 0.16g, 0.32g and 0.48g respectively. The base acceleration and the computed surface acceleration for D_r = 45% and a_{max} = 0.32g are shown in Fig. 4.

For the purpose of analysis the layer was divided into 10 slices each 5ft (1.52m) thick. The dynamic shear stresses and associated shear strains for a typical slice are shown in Figs. 5a and 5b. The volumetric strains generated by the strain history in Fig. 5b were computed using equation (1) and are shown in Fig. 5c. In a horizontal layer of sand these strains are equal to the vertical strains and when multiplied by the thickness of the slice give the total settlement of this slice as a function of time. When similar results for other slices are summed the distribution of settlements throughout the depth of the layer are obtained as a function of time.

The distribution of total settlements under shaking by the N-S component of El Centro (1940) with maximum acceleration of 0.32g is shown in Fig. 6 for relative densities D_r = 45%, 60%, and 80%. The dependence of settlement on relative density is obvious. It should be noted that even very dense sands (D_r = 80%) can undergo appreciable settlement under strong shaking. In the example shown the settlement was of the order of 0.5in (1.3cm) when the relative density D_r = 80%.

The total settlement in a given sand stratum depends on the frequency content and magnitude of the base input motion. Since the motion is fed in at bedrock or an equivalent stiff layer the frequency content should be rather high. This kind of frequency content is represented reasonably well by the El Centro record. If necessary the record may be scaled to higher frequencies.

The effect of variations of the maximum accelerations are shown as functions of the relative density in Figure 7. The non-linear nature of the response of the sand is clearly evident.

The settlement of a sand will be affected by the existence of a structure founded on the sand. The weight of the structure increases the mean normal effective stress in the sand and so increases the shear modulus G, in accordance with equation (3). The increased values of G lead to increased resistance to shearing strain and hence to volume change. This effect tends to reduce the total settlement. However, the mass of the building has an opposite effect. During an earthquake the base shear generated in the building by inertia forces generates additional shear stresses and strains in the sand and tends to increase settlements. The net effect of these opposite influences in all cases studied was to increase the total settlement in the sand. Thus the free field settlements of a sand stratum provide a lower bound to the settlements to be expected under a structure during an earthquake. This conclusion is quite important because of the difficulty of determining settlements in a combined soil-structure interaction analysis.

The effect of a structure on settlements may be modelled by adding an extra slice to the discrete mass system with a very high stiffness and a mass sufficient to cause foundation pressures ranging from 2000 p.s.f (1 kg/cm^2) to 8000 p.s.f. (4kg/cm^2). The distribution of settlements with depth caused by the first 10 secs of El Centro (1940) are shown in Fig. 8 for both the free field case of no surcharge (σ =o) and for a building causing a surcharge of σ = 4000 p.s.f. (2 kg/cm^2). The relationship between surface settlement and surface load for a range of relative densities is shown in Fig. 9. The sharp curvature in the D_r = 45% curve at the higher surface pressures is due to a significant shift in the fundamental period of the sand stratum at this relative density due to the surface pressures and superimposed mass. It seems that the presence of a structure leads to considerably higher settlements during ground shaking.

In all the solutions analysed above the sand was assumed to be of uniform relative density. Sand deposits in the field are rarely uniform. Site investigation usually reveals variations in density in both the vertical and horizontal directions. Variation in density in the vertical direction presents no difficulty to the computation of settlements. The shear slices are selected for the discrete mass model in such a way that density variation within the sand depth modelled by each slice is small.

Consider, for example, the case in which it is decided to minimize static load settlements by compacting the upper half of the stratum of sand having D_r = 45% so that the top 25ft (7.62m) is now at D_r = 80% and the bottom 25ft (7.62m) is at D_r = 45%. The distribution of settlements with depth under a surface pressure of 2 kg/cm^2 is shown in Fig. 10 for shaking by the N-S component of El Centro (1940). The settlements that would occur if the layer had a uniform density D_r = 45% is shown also for comparison. It is evident that most of the settlements are in the underlying looser portion of the deposit. The compaction of the upper levels of a loose deposit may be effective in reducing static settlements but may prove ineffective in preventing significant settlement during an earthquake.

When the shear properties vary in the horizontal direction the shear beam analysis is no longer theoretically exact because an underlying assumption of the method is that the sand is uniform in the horizontal direction. Consider two locations in a sand stratum 30ft (9.15m) apart and let the densities at the locations be D_r = 45% and D_r = 60%. The surface settlements at these locations during shaking by El Centro (1940) are, from Fig. 6, 1.94 in. (4.92 cm) and 1.14 in. (2.88 cm). Since the sand is not uniform each of these computed values is somewhat in error and except in the neighbourhood of sharp discontinuity in properties the percentage magnitude of the error is likely to be about the same in each case. Thus the error in the computation of the differential settlement, which is the difference between these values, is likely to be very much smaller. It is our opinion that differential settlements may be computed with sufficient accuracy for practical purposes using the method outlined above.

Conclusions

A method has been presented for estimating the settlement of strata of unsaturated sand during an earthquake when the sand is uniform in a horizontal direction. These uniform settlements will not cause structural damage but an estimate of their magnitude may be important in low-lying areas where thewater table is close to the surface and subsidence could cause flooding.

When conditions vary in the horizontal direction the method may be used to esimate the differential settlements as well as the total settlement. In this case the differential settlements are likely to be more accurate than the total settlements.

Differential settlements can occur not only because of a variation in soil properties but also because of different pressures and masses superimposed on the sand layer by the foundations of structures. The proposed method can take these masses and pressures into account in the computation of settlements.

Compaction of the upper levels of a loose deposit of sand may be effective in reducing the settlements due to static load but may be quite ineffective in reducing the settlements due to earthquake loading. This is because the zone of significant settlement for static loading is within a depth equal to that of the smallest dimension of the loaded area whereas for earthquake loading significant settlenents take place within the full depth of the deposit.

The results of the analyses of many cases show that the relative density of the sand, the time history of horizontal acceleration, and the surface masses and pressures are the primary factors affecting the settlements. Vertical accelerations do not generate significant settlements.

The procedure and results described above are directly applicable to saturated sands provided the porewater pressure does not increase beyond 30% of the effective overburden pressure. For higher porewater pressures the possibility of a foundation failure due to liquefaction or high pore pressure becomes more likely and a different kind of analysis is necessary.

Acknowledgements

The research described above was supported by the National Research Council of Canada under grants NRC 1498 and NRC A5109.

Notation

The following notation has been used in the paper.

[C] = damping matrix

D_r = relative density in per cent

G = shear modulus

H = stratum thickness

h_i = thickness of i^{th} slice

[K] = stiffness matrix

K_2 = shear modulus factor

k_i = stiffness of i^{th} spring

[M] = diagonal mass matrix

R = conversion factor for volumetric strain

$\{x\}$ = displacement vector

$\{\dot{x}\}$ = velocity vector

$\{\ddot{x}\}$ = acceleration vector

γ = shear strain in per cent

$\Delta\varepsilon_{vd}$ = increment in volumetric strain in per cent

ε_{vd} = volumetric (and vertical) strain in per cent

λ_i = critical damping ratio in i^{th} mode

σ = surcharge pressure

σ'_m = mean effective normal stress

$[\phi]$ = mode shapes

ω_i = i^{th} natural frequency

References

1. D'Appolonia, E., (1968). "Dynamic Loading," State of the Art Report presented at the August 1968 ASCE Speciality Conference held at Cambridge, Mass.

2. Finn, W.D. Liam and Byrne, P.M. (1975). "Seismic Response and Liquefaction of Sands". (submitted for publication) Soil Mechanics Series No. 24, University of British Columbia, Vancouver.

3. Lee, K.L. and Albaisa, A., (1974). "Earthquake Induced Settlements in Saturated Sands", Jour. of the Geotechnical Division, ASCE, Vol. 100, No. GT4, April 1974, pp. 387-405.

4. Martin, G.R., Finn, W.D. Liam, Seed, H. Bolton, (1975). "Fundamentals of Liquefaction under Cyclic Loading", (to be published in Journal of the Geotechnical Division, ASCE in May 1975). Soil Mecahanics Series, No. 23, Univ. of British Columbia, Vancouver.

5. Polshin, D.W., and Tokar, H.A. (1957). "Maximum Allowable Settlement of Structures." Proceedings, 4th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, p. 402.

6. Seed, H.B. and Idriss, I.M. (1969). "Influence of Soil Conditions on Ground Motions During Earthquakes," Jour. of the Soil Mechanics and Foundations Division, ASCE, Vol. 95, No. SM1, January 1969, pp. 99-137.

7. Seed, H.B. and Idriss, I.M. (1970). "Soil Moduli and Damping Factors for Dynamic Response Analyses". Report No. EERC 70-10, Univ. of Calif. Earthquake Eng. Research Center, Berkeley, December 1970.

8. Seed, H. Bolton and Silver, M.L. (1972). "Settlement of Dry Sands during Earthquakes", Jour. of the Soil Mechanics and Foundations Division, ASCE, Vol. 98, No. SM4, April 1972. pp. 381-397.

9. Silver, M.L. and Seed H.B., (1971). "Volume Changes in Sands during Cyclic Loading" Jour. of the Soil Mechanics and Foundations Division ASCE, Vol. 97, No. SM9, Sept. 1971, pp. 1171-1182.

10. Skempton, A.W., and MacDonald, D.H. (1956). "Allowable Settlement of Structures". Proceedings 4th International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, p. 402.

11. Whitman, R.V., and Ortigosa, P.O. (1969). "Densification of Sands by Vertical Vibrations" Proceedings of the 4th World Conference on Earthquake Engineering, Chile, 1969.

12. Youd, T.L. (1970). "Densification and Shear of Sand during Vibration". Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 96, No. SM3, May 1970, pp. 863-880,

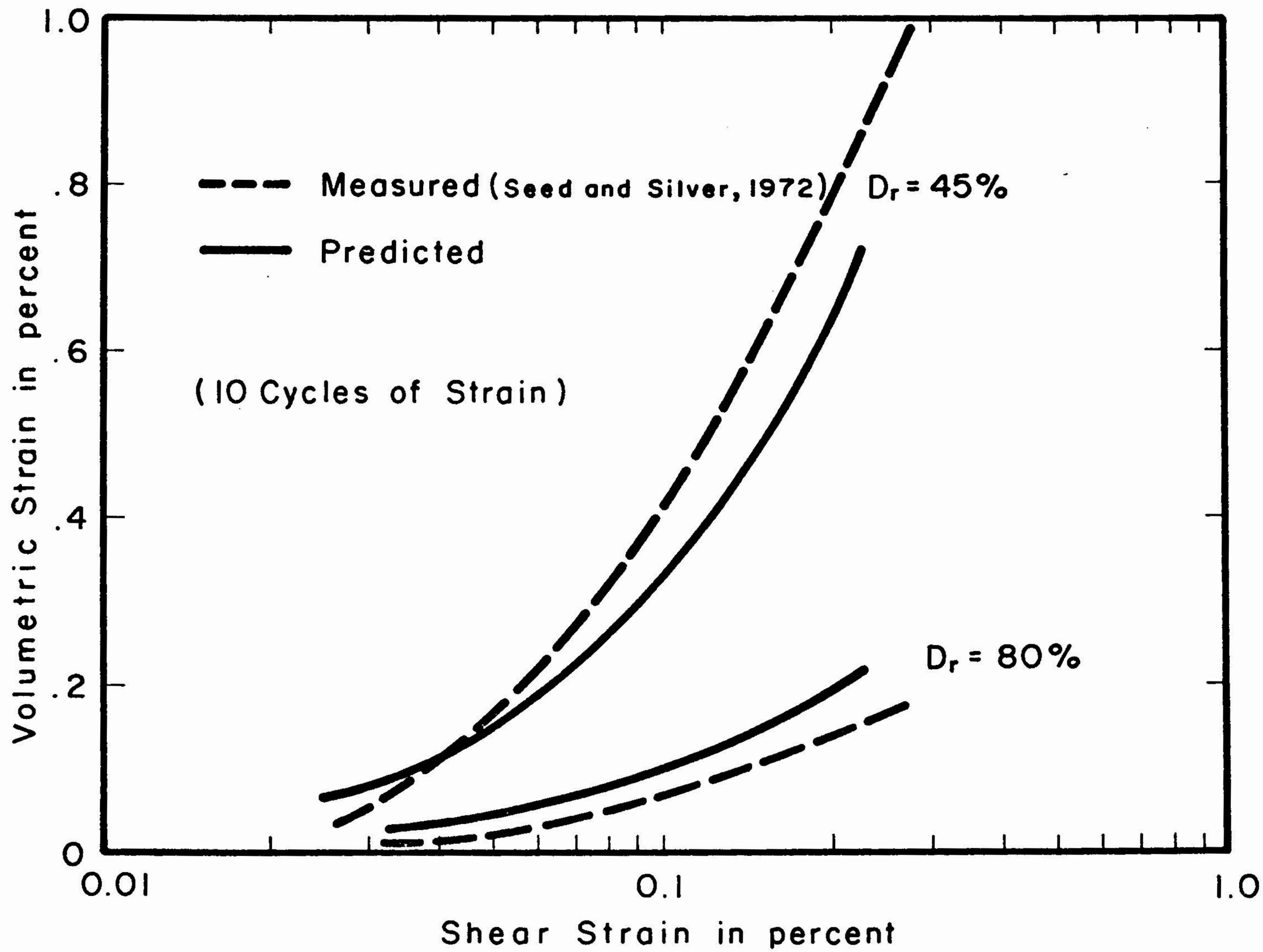


FIG. 1 COMPARISON OF COMPUTED AND MEASURED STRAINS.

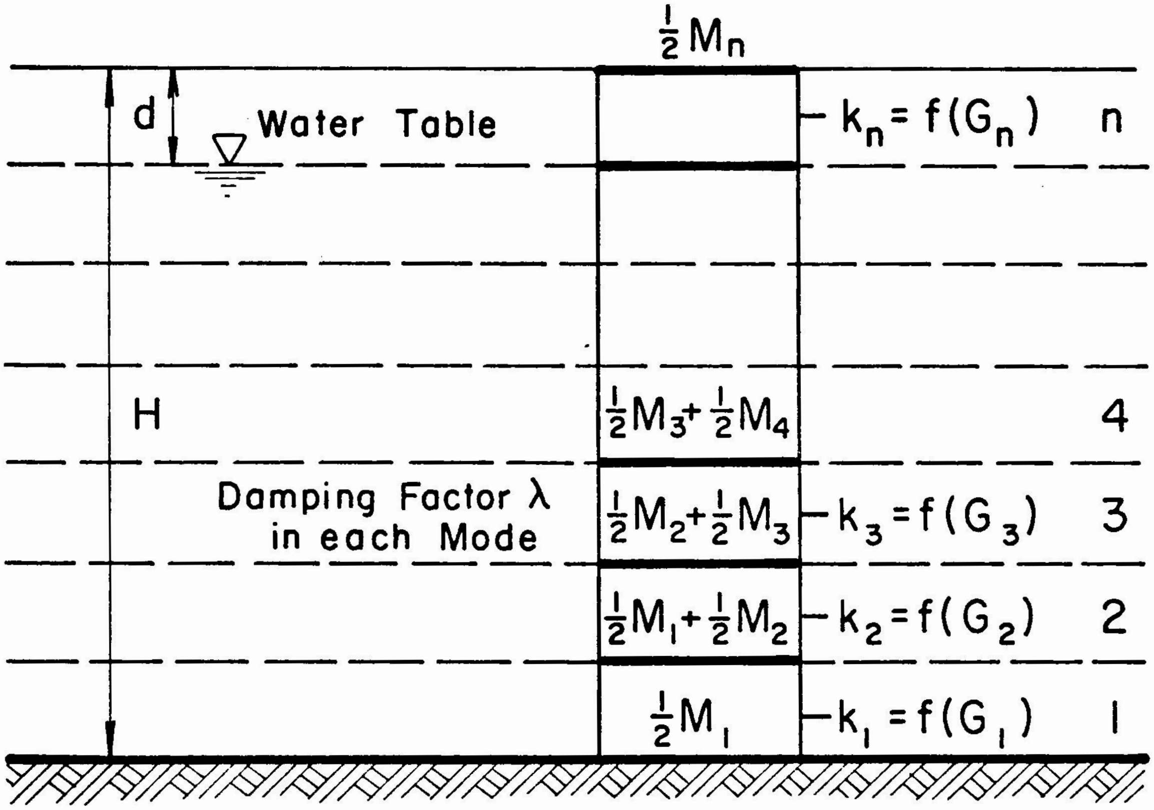
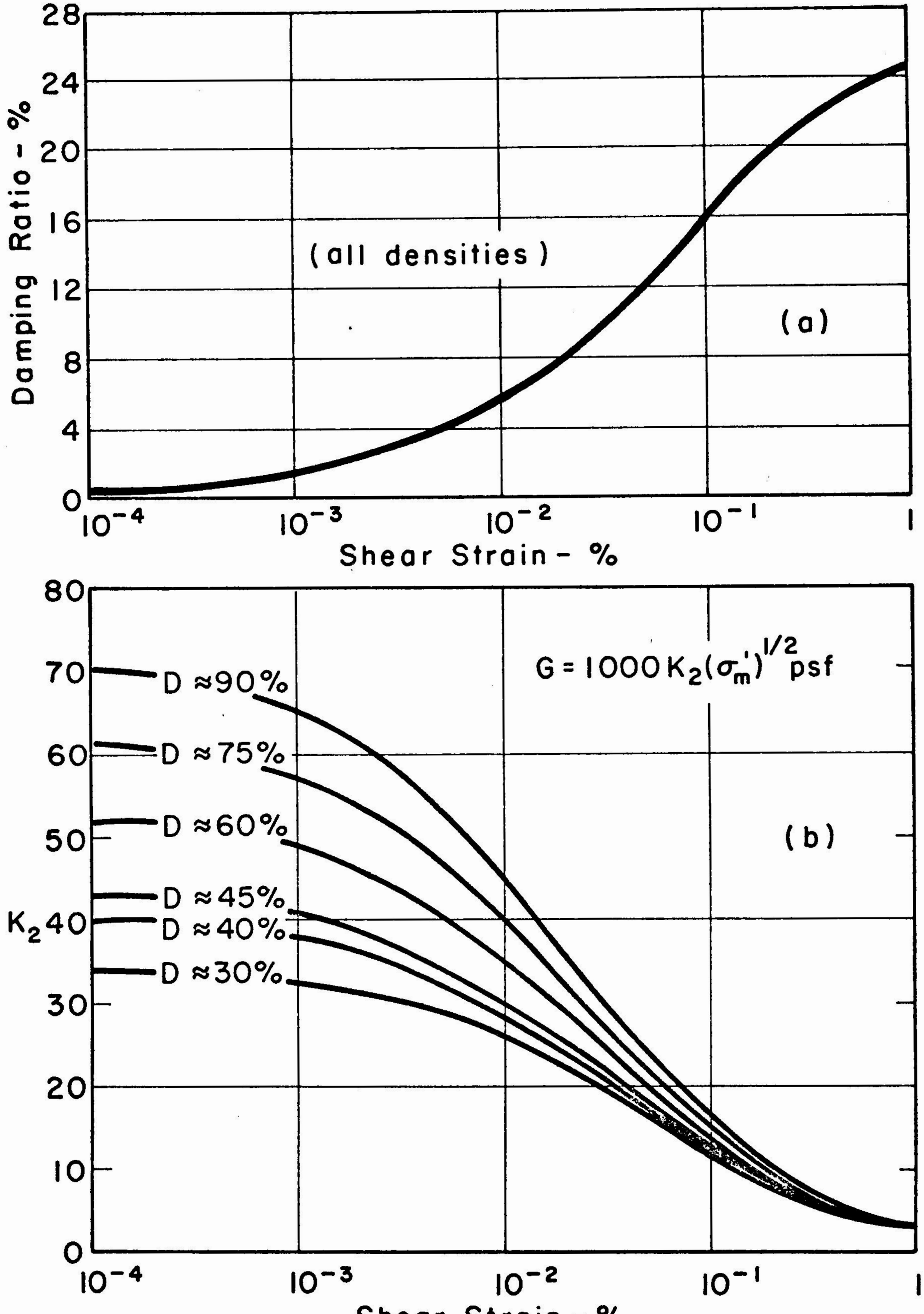


FIG. 2 DISCRETE MASS MODEL OF FOUNDATION LAYER.



(From Seed and Idriss, 1970)

FIG.3 MODULI AND DAMPING RATIOS FOR SAND.

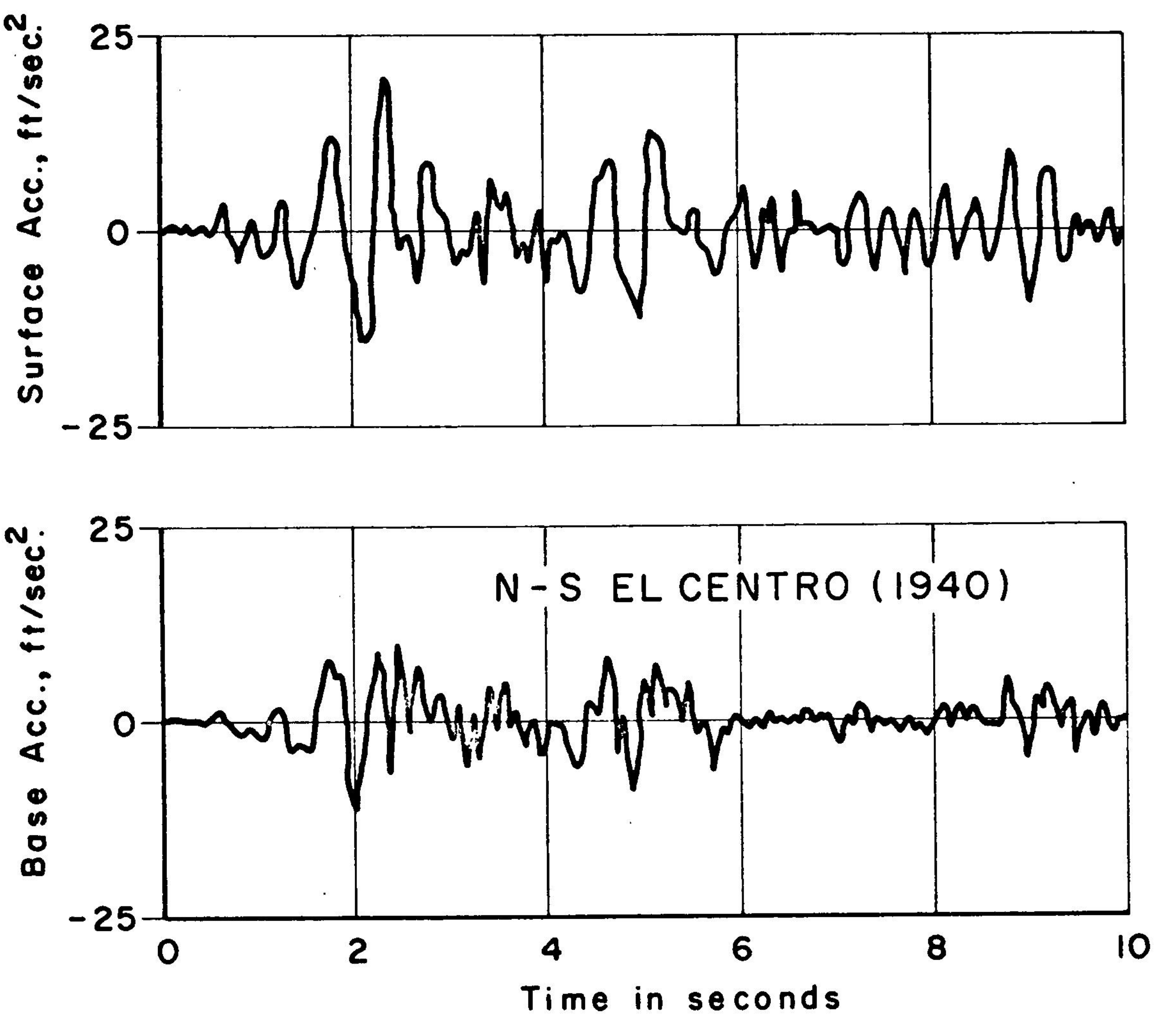


FIG. 4 BASE AND SURFACE ACCELERATIONS.

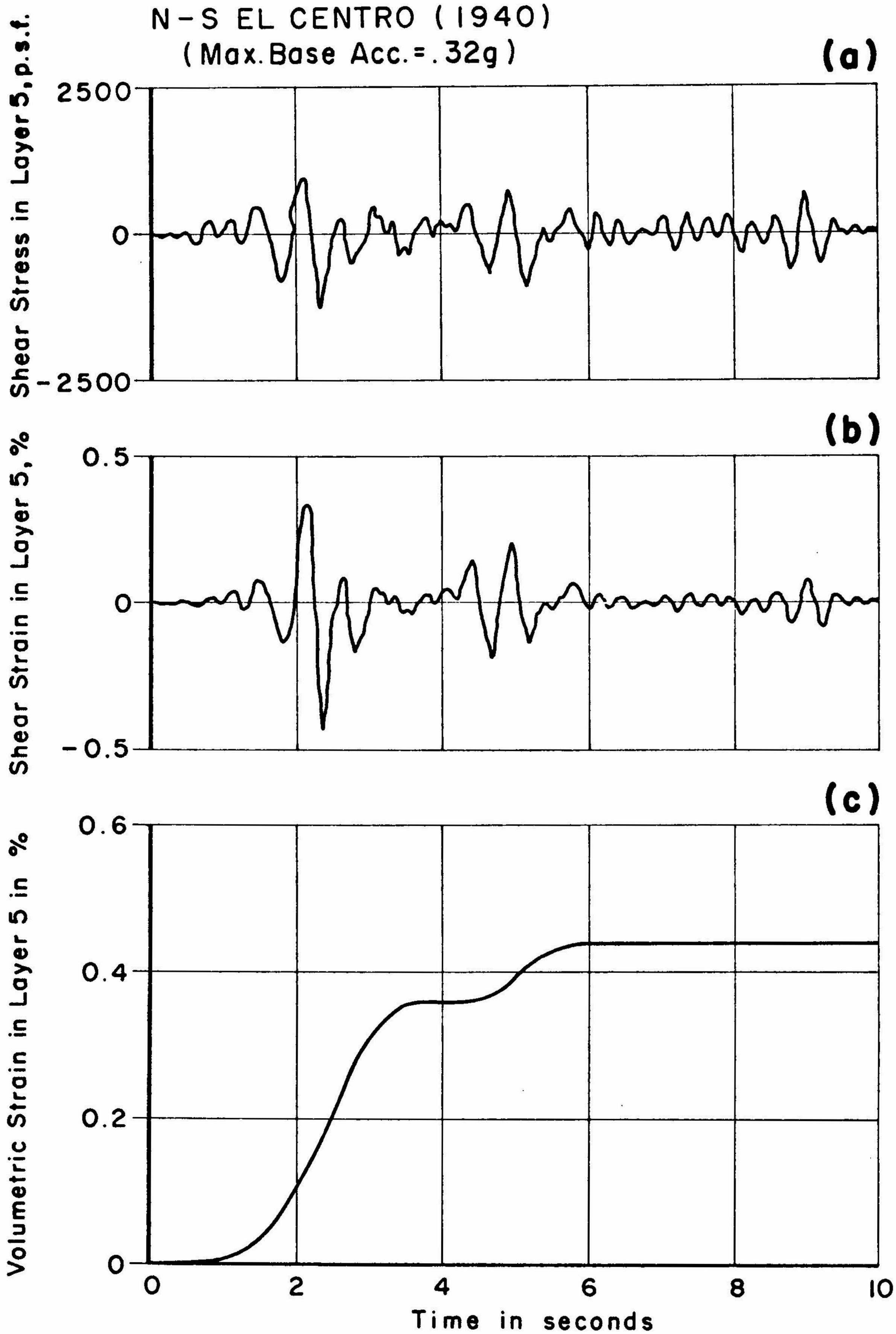


FIG. 5 DYNAMIC STRESSES AND ASSOCIATED STRAINS.

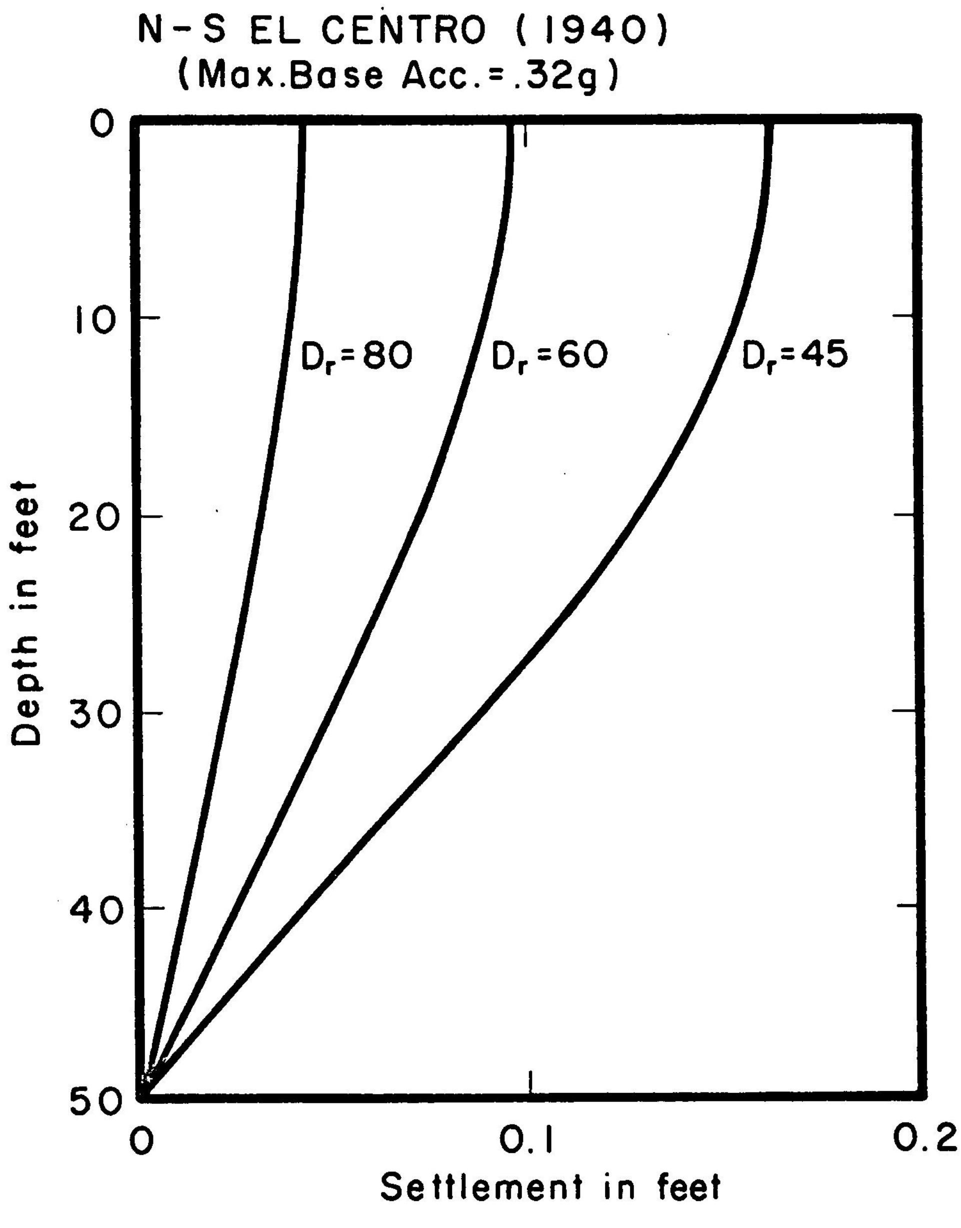


FIG.6 SETTLEMENT VS. DEPTH FOR VARIOUS DENSITIES.

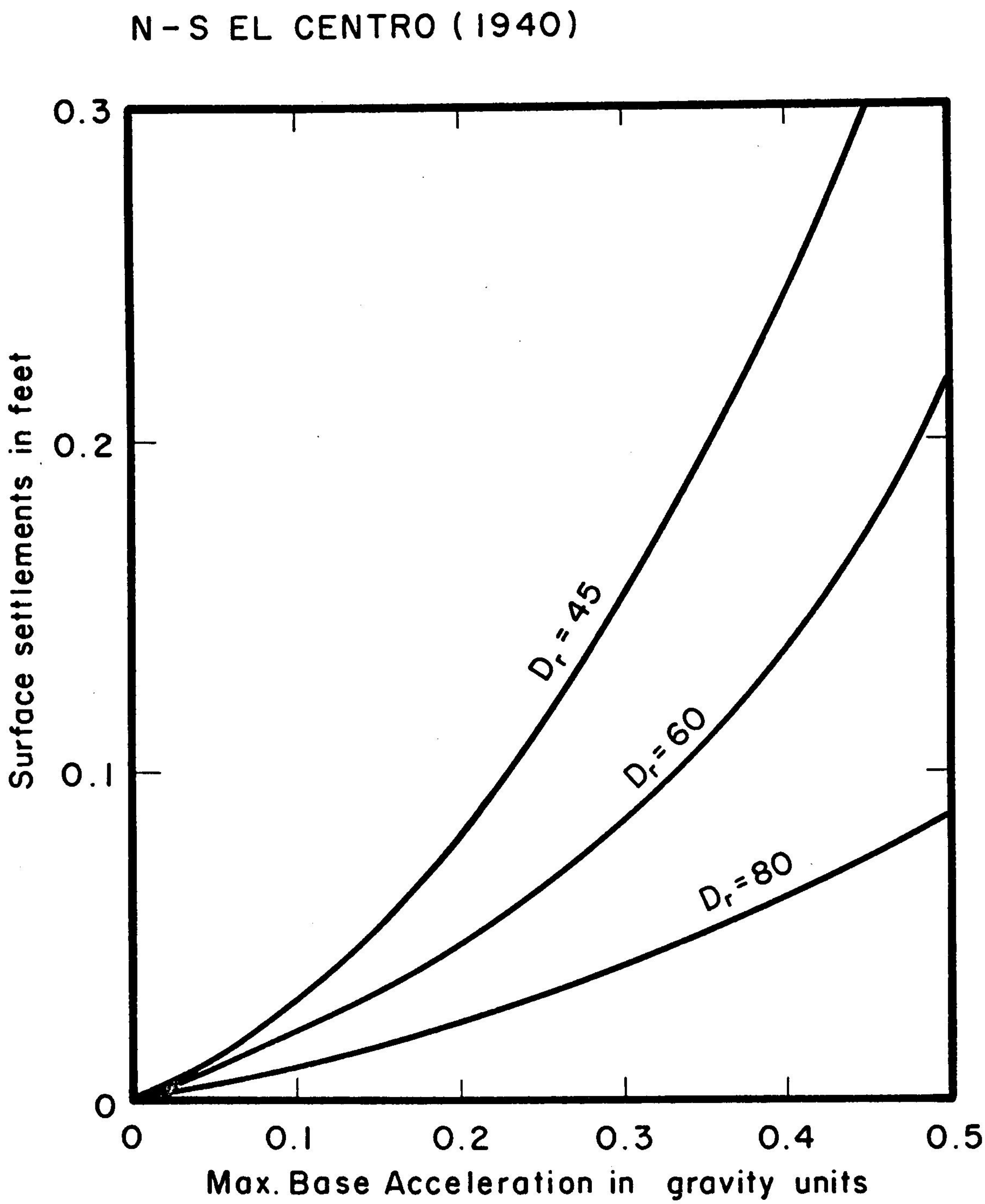


FIG. 7 SURFACE SETTLEMENTS VS. MAXIMUM BASE ACCELERATION.

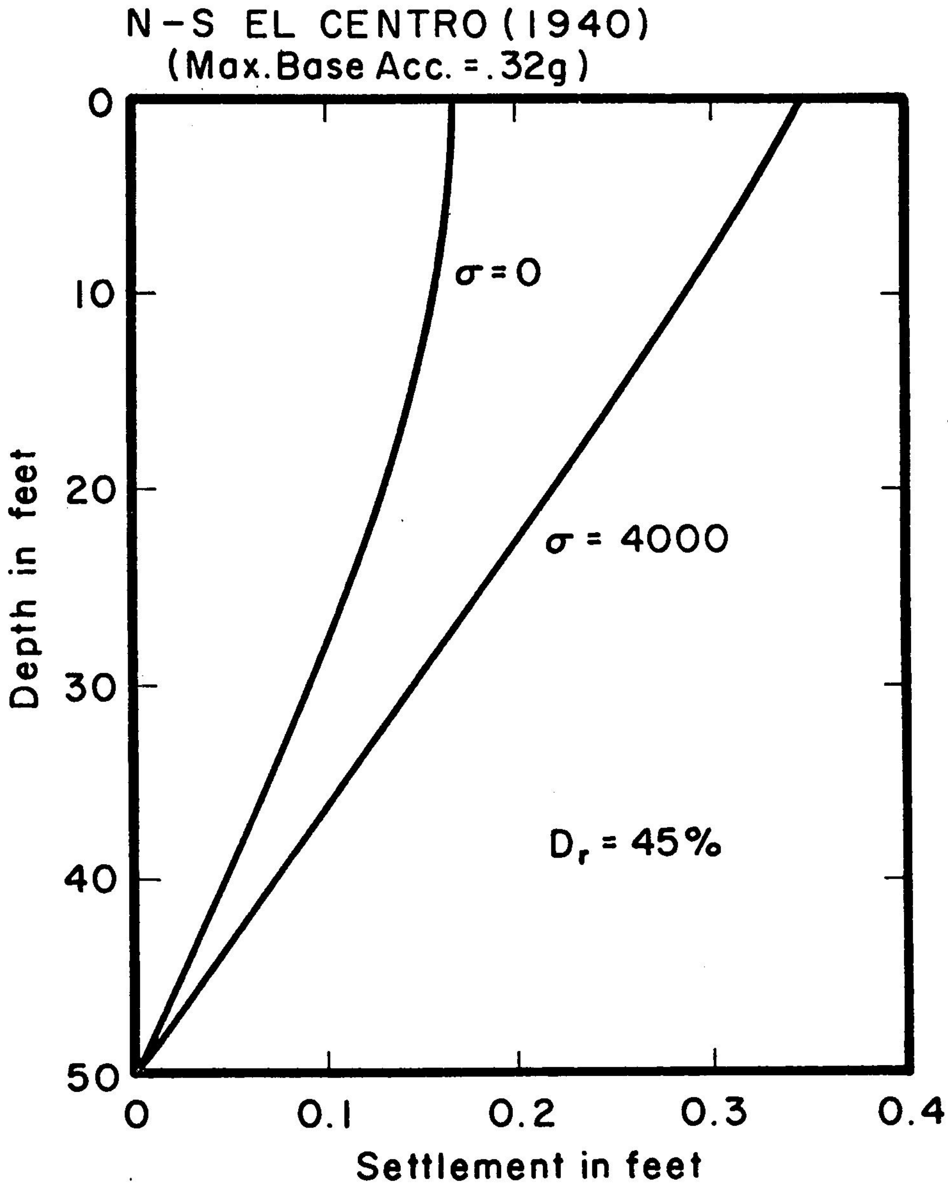


FIG.8 EFFECT OF SURFACE LOADS ON SETTLEMENTS.

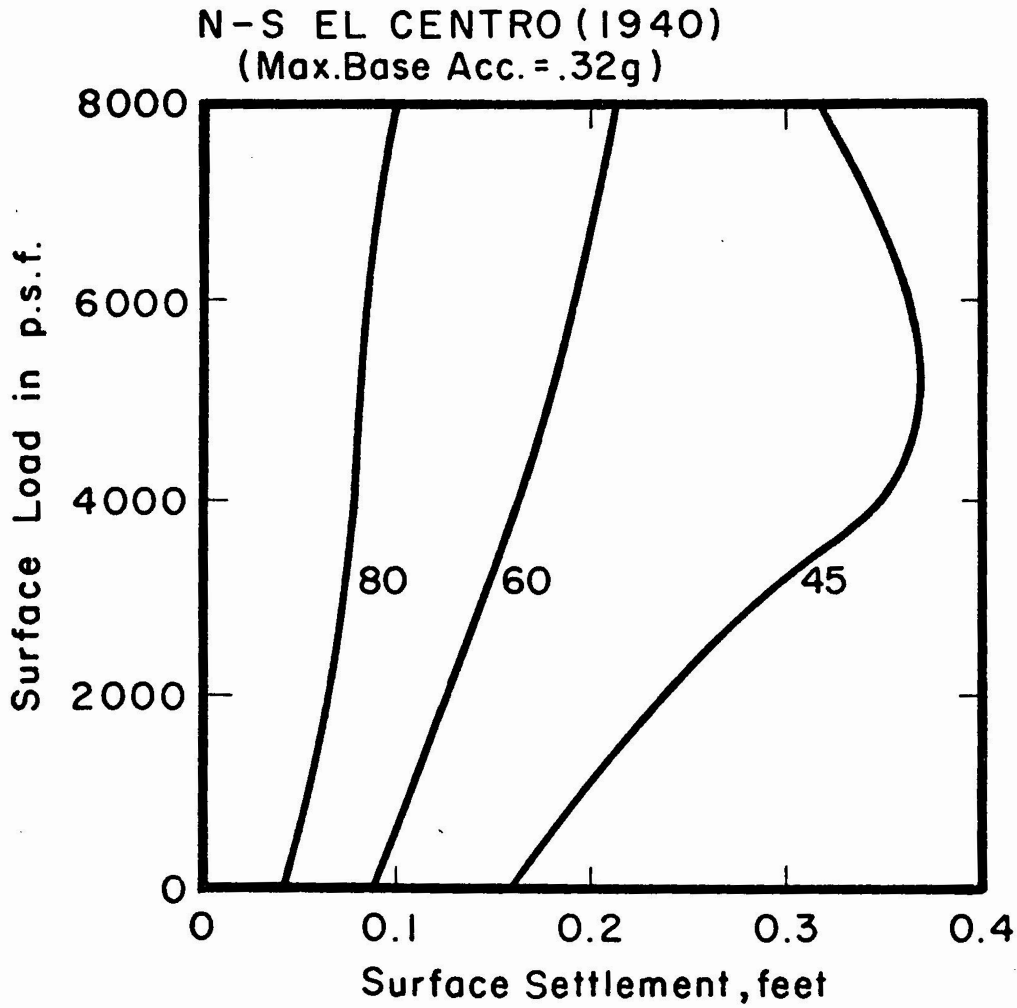


FIG. 9 EFFECT OF SURCHARGE AND DENSITY ON TOTAL SETTLEMENT.

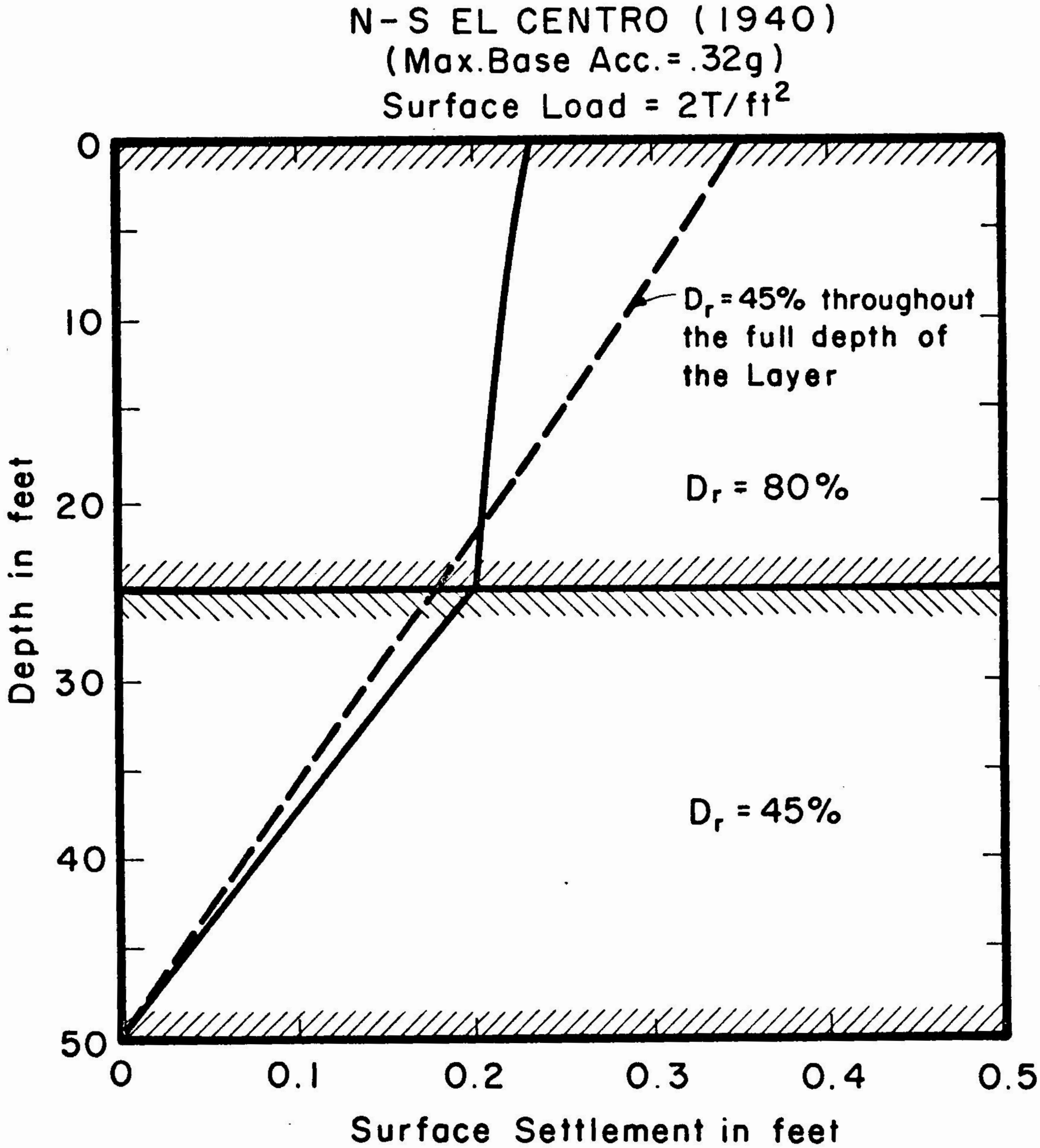


FIG.10 SETTLEMENT DISTRIBUTION IN NON-UNIFORM SAND.